Matching Loop Antennas To Short-Range Radios

The tapped or transformer-matched loop antenna must be properly matched to differential drives in many short-range radio designs to achieve optimum performance.

Short-range radios are invaluable wireless systems for data, telemetry, and voice communications. But to achieve optimum operation, the radio electronics must be properly matched to the antenna. Part 6 of this design series on short-range radios will address the design of tapped or transformer-matched loop antennas and show how they can best be matched to differential drives. The final installment of this multipart series will appear later this year and will cover practical issues in board design such as layout, shielding, cost control, and regulatory compliance.

Previously, Parts 1, 2, 3, 4, and 5 of this series (see *Microwaves & RF*, September and October 2001 and February, March, and July 2002, respectively) explored short-range radio design, including link budgeting, regulatory issues, device fabrication, and loop-antenna design. Part 5 offered an introduction to loop-antenna design and the tapped-capacitor-matching method. This article will provide the basic theoretical base to understand the inductively tapped-loop antenna, which requires a lower part count than other methods, always a popular feature in the very cost-constrained short-range-radio world. A modeling method that enables understanding and design of differentially driven loop antennas is also shown. Differential drive is popular in integrated-circuit (IC) transmitters (Txs) since it aids stability in the presence of bond wire and pin inductance, provides some degree of immunity to power supply and ground noise, and can provide higher output power in the case of

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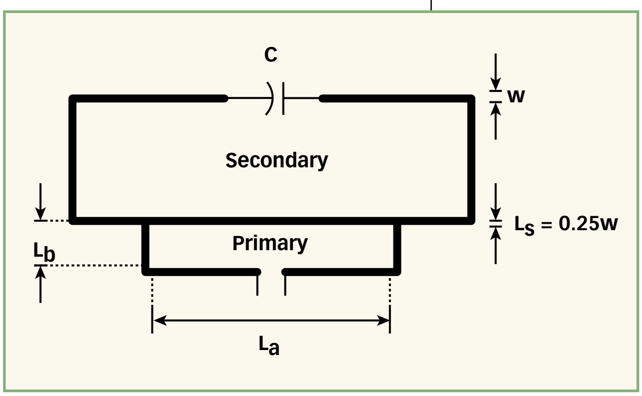
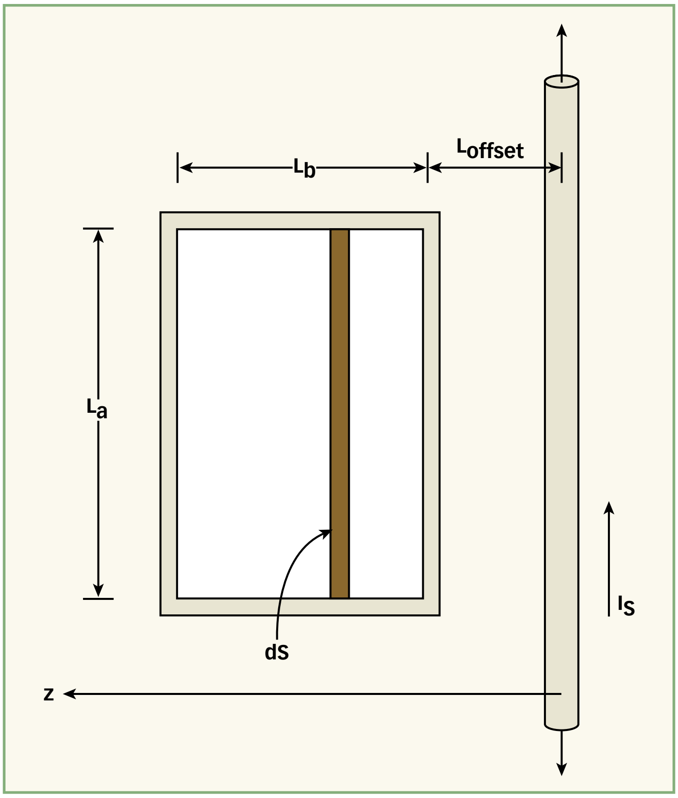


13. The typical printed-circuit-board (PCB) implementation of the transformer matched loop antenna is shown here.

some device limits.

Although the term "tapped loop" is common, this type of antenna will be referred to here as the "transformer" loop antenna in reference to what is actually its fundamental mode of operation. Understanding this matching method requires resorting to the underlying electromagnetics (EMs). Once the new model is grasped it leads directly to understanding the harmonic performance of the tapped/transformer loop antenna.



14. Setting up the integration of flux density that enables calculating the mutual inductance between a closed primary and a wire secondary is illustrated here.

As shown in **Fig. 13,** a small loop is placed near (usually actually sharing a side with) the radiating loop antenna. The radiating loop still contains a tuning capacitor C. The two loops actually form a "loosely coupled" transformer, though there is a strong tendency among circuit designers to want to view this structure as a tapped inductor (no mutual coupling) or autotransformer (tapped inductor with mutual coupling). The correct model will be shown here to be a separated transformer, though understanding this will require a bit of an effort on the part of the reader. The transformer model seems counter-intuitive, even to experienced RF designers, since they are trained to think in lumped-component terms and not in terms of the underlying EMs upon which lumped models are based. Thus they normally conceive of a segment of trace as having complete inductance all by itself in the absence of a return path, which leads them to misinterpret Fig. 13 as a tapped inductor or autotransformer. No less an authority than Fujimoto[9] in his well-respected work on small antennas mistakenly analyzes tapped-loop antenna matching as an autotransformer, and this common error incorrectly influences the design of loop antennas to this day. The mistaken mental model has at its root the failure to understand that only closed current loops have inductance or mutual inductance. It is exacerbated by the fact that the form of transformer exhibited by Fig. 13 is not one that the engineer has encountered in his basic training—no class ever showed a separated transformer model for a situation where primary and secondary currents actually share a path segment.

An open mind and a review of the underlying EMs will allow the short-range radio designer to add this important form of transformer antenna to his tool kit and gain an appreciation of the EM effects in circuits that the designer's first EMs professor probably intended. To set about developing the correct first-order understanding of this structure, the authors shall state the basic EMs upon which transformer-model argument is based with minimal explanation, leaving the reader to review their basic undergraduate e-mag text for verification. However, the authors will interpret these EMs with respect to this new situation, the loop antenna of Fig. 13, in some detail to make the model fully clear and generate the correct mental model in the reader's mind.

First consider, as background, the definition of a voltage [as electromotive force, (EMF)] as the closed line integral of electric field, which is the field form of Kirchoff's Voltage Law (KVL):

$$emf = \oint \vec{E} \bullet \vec{dL} \quad (71)$$

Next, consider the fact that electric flux through a surface is provided as the surface integral of flux density over that surface:

$$\Phi = \oint_S \vec{B} \bullet \vec{dS} \quad (72)$$

Flux is integrated up over an area—not over a line segment. Next, Faraday's Law offers voltage (EMF) as a function of flux:

$$emf = \frac{d\Phi}{dt} \quad (73)$$

Comparing Eq. 71 and 73, we note that the voltage around a loop is equal to the negative of the time derivative of flux through the loop.

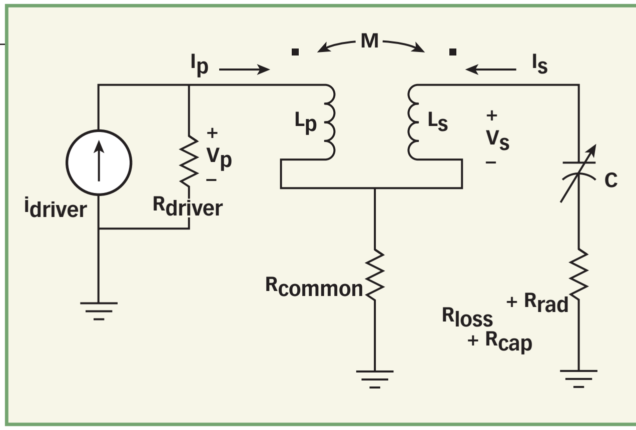
Ampere's Law gives current as the closed-line integral of magnetic field:

$$I = \oint \vec{H} \bullet \vec{dL} \quad (74)$$

Where magnetic field H is related to flux density B by:

$$\vec{B} = \mu_0 \vec{H} \quad (75)$$

When terminal voltage and current can be calculated and impedance determined as their ratio, a circuit model results. The EM equations above provide the means to determine current and voltage relationships in terms of physical geometry. Eq. 74, Ampere's Law, relates flux and current over a closed line integral that provides current con-



15. The circuit model of a transformer-matched loop antenna acts as a separated transformer with the minor exception of the shared resistance over the common section of trace.

tained within the closed-path caused by flux. From Ampere's Law, current can be found from H or B, or H and B can be found from current. When B is known, the total flux can be found from Eq. 72, and then with flux known, voltage can be found from Eq. 73. Conceptually, the full information needed for the circuit model is available, and from Eqs. 71 to 74 it can be seen that this always relies upon closed paths around current or field, and not upon a line segment. Alternately, the definitions of inductance and mutual inductance provided by Eqs. 76 and 77 can be used to make this conceptual process a bit shorter:

$$L = \frac{N\Phi}{I} \tag{76}$$

$$M_{12} = \frac{N_2\Phi_{12}}{I_1} \tag{77}$$

where:

N = the number of filamentary loops of current (one in Fig. 13) and

I = the current "linked" by the flux, meaning the current that surrounds the area the flux density is integrated over to get the total flux.

In Eq. 77, M_{12} is the mutual inductance where flux produced by closed (or infinite) path I_1 links current in closed or infinite path I_2. It is also true that $M_{12} = M_{21}$. Parameters L and M result in circuit equations of the form:

$$V_1 = L\frac{dI_1}{dT} + M\frac{dI_2}{dT} \tag{78}$$

where:

V_1 = the total voltage through a self-inductance with current I_1 that is also linked to a second current I_2 sharing mutual inductance M with the current path described by I_1.

Note that in Eqs. 75 and 76 inductance cannot be calculated for a segment of line. It requires a closed path around a surface to obtain the total flux quantities as the surface integral of flux density. This is why a tapped inductor or autotransformer model of Fig. 13 is simply wrong—it does not satisfy the definition of inductance. But an integration over a closed surface, such as the primary and secondary shown in Fig. 13, gives total flux linking a closed current path, which then by Eqs. 76 and 77 allows calculation of self- and mutual inductance that enables writing circuit equations.

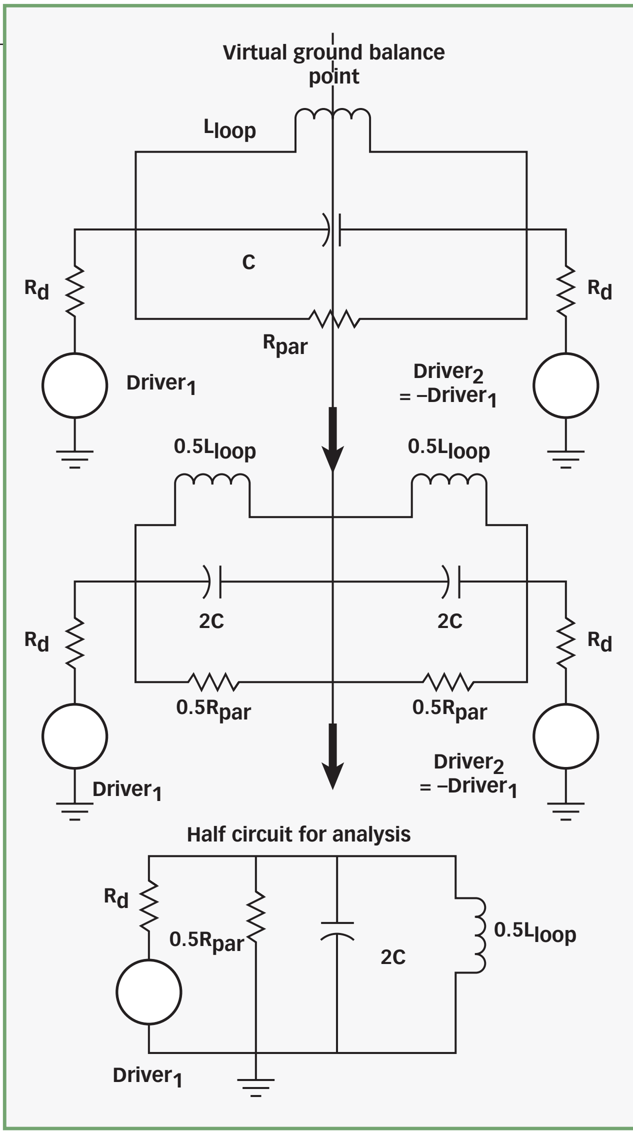
With the preconceived circuit-design model altered to take these fundamentals into account, it is now possible to find a correct (transformer-based) circuit model for Fig. 13. **Figure 14** shows a loop intended as the primary winding of inner dimension L_a and L_b linked by the flux generated by an infinitely long, thin, round wire. The loop is also considered to be made of thin round wire and its inner dimension is separated from the center of the infinite wire by distance L_{offset}. Of course, most antennas will be fabricated with printed-circuit-board (PCB) materials having a flat trace, but the round wire model is simpler analytically and is a good approximation of an antenna formed of circuit traces, and so is used here. Most basic EM texts go through the small exercise needed to use Ampere's Law (Eq. 74) to obtain radial H and B fields around the infinite round wire induced by current I_s in the wire. This yields:

SEE EQ. 79 ON P. 79

Using Eq. 72 and the differential area element dS shown to integrate over the area of the primary (L_a X L_b), a few lines will yield the flux, and then dividing by I_s as per Eq. 77 yields the mutual inductance:

SEE EQ. 80

Equation 80 is a useful accurate approximation (slightly large) of the mutual inductance between a small primary winding and large secondary winding, as the other sides of the secondary are much farther away from the primary winding. The separated form as shown may be used if the maximum possible mutual inductance is not needed (it will be shortly shown how impedance is controlled by mutual inductance). If maximum mutual inductance is desired, the two loops may be brought into actual contact, at which point L_{offset} will be equal to the radius of the secondary wire plus the diameter of the primary wire (not zero, which would be unacceptable in the denominator in Eq. 80). When the two loops are brought into contact, there will be no drastic change in the circuit model, which is the tricky point for most circuit designers to accept. The only effect that contact has on the model is to force the primary and secondary currents to mix in the shared segment, but this does not change the fundamental nature of the structure giving the mutual inductance which dominates the behavior. When the currents are shared in the segment, there is a small voltage induced in the primary and secondary coils due to resistance in the shared segment, not only from each on its respective side, but also from the other. For the best possible accuracy this leads to the technical need for the model to have either a single resistor in the common (to ground) terminal of primary and secondary coils, or for a "trans-resistance" to be inserted in each of the primary and secondary coils. It is critically important to note that the contact does not force



16. The half-circuit concept provides an understanding of the application of single-ended drive analysis to differentially driven loop antennas.

$$\vec{B} = \frac{\mu_0 I_s}{2\pi z} a_\phi \qquad (79)$$

$$M = \frac{\Phi}{I_s} = \frac{\mu_0 L_a}{2\pi} 1n\left(1 + \frac{L_b}{L_{offset}}\right) \qquad (80)$$

an autotransformer model. The shared segment is not an inductor, only the complete current loops of primary and secondary are true inductors. An autotransformer model would be appropriate only if a loop of primary were drawn inside the secondary.

$$I_p\left(j\omega L_p + R_p\right) + I_S j\omega M = V_p \quad (81)$$

$$I_p j\omega M + I_S\left[j\left(\omega L_S - \frac{1}{\omega C}\right) + R_S\right] = 0 \quad (82)$$

$$Z_{IN} = \left(R_P + \omega^2 M^2 \frac{R_S}{R_S^2 + X_S^2}\right) + j\left(X_P - \omega^2 M^2 \frac{R_S}{R_S^2 + X_S^2}\right) \quad (83)$$

Figure 15 shows the desired circuit model for the transformer-matching case using the concepts developed earlier. The small loop is designated as the primary and the large loop as the secondary of the transformer. Despite the fact that the loops of Fig. 13 are touching and share a side, the structure truly functions as a separated transformer with the exception that the shared side has a loss and radiation resistance that is represented as R_{common}. Normally, this common resistance is so small that it may be set to zero in calculations.

Figure 15 does not show a suitable transformer with infinite inductance and winding ratio N that yields impedance transform N^2. It is a linear transformer of winding ratio one for which full circuit equations must be written. Neglecting R_{common}, we may write primary and secondary KVL equations as:

SEE EQ. 81 ABOVE
SEE EQ. 82 ABOVE

Solving this equation set for primary voltage and current and then taking their ratio as input impedance yields:[15]

SEE EQ. 83 ABOVE

where:

Z_{IN} = the total complex input impedance,
X_P = the magnitude of the reactance of the primary,
X_S = the magnitude of the reactance of the secondary, and
M = the transfer inductance between the two loops (in Henrys).

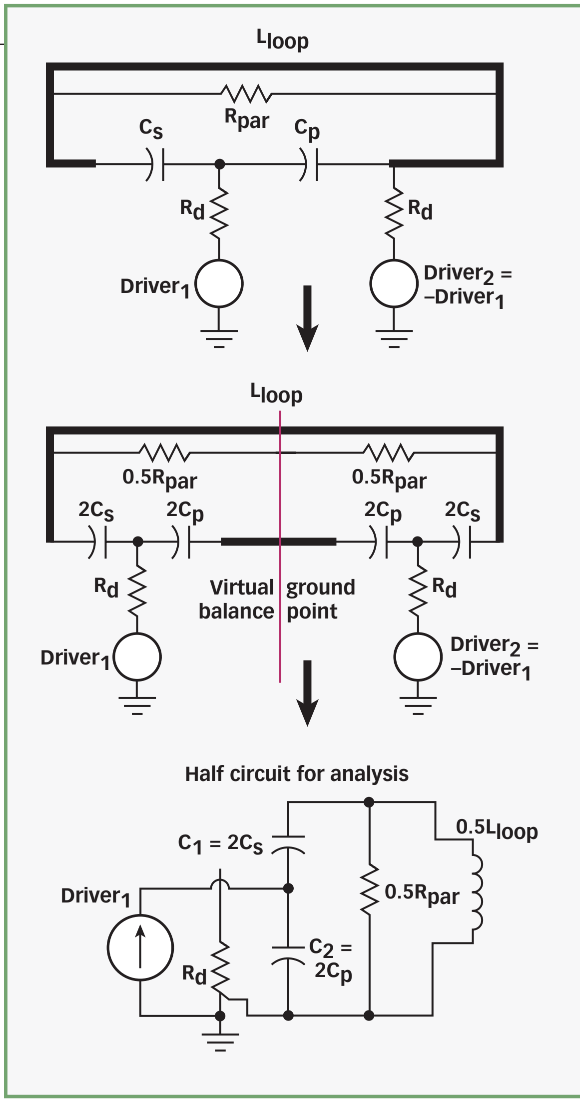
From Eq. 83, it should be noted that when X_S is zero (secondary resonance), Z_{IN} still contains some reactance from the primary inductor impedance X_P. In practice, an extremely small amount of positive (inductive) secondary reactance is required (for example, by varying the loop capacitance slightly so that secondary resonance is slightly below the operating frequency) to obtain a purely real Z_{IN}. From Eq. 83, it is possible to find that the input impedance at resonance as approximately:

SEE EQ. 84 BELOW

Equation 84 provides a simple method to match the low series resistance R_S of a resonant loop antenna to the high impedance (kohms) required by complementary-metal-oxide-semiconductor (CMOS) ICs. Initially, Eq. 84 is used to calculate the needed transfer inductance M to achieve a specific input impedance $Z_{IN} = R_D$ for matching. Secondly, using Eq. 80, L_a, L_b and offset are adjusted until the required M is achieved. Eq. 80 will generally be found to be accurate within about 10 percent, but if the greatest possible accuracy is desired, an EM simulator can be used to refine the geometry more closely. As will be seen later, to minimize radiation from the primary loop and to lower primary loop reactance, L_a should be made as large as possible, and L_b and offset should be made as small as possible.

We may now use this model of the transformer loop antenna to predict harmonic performance at frequencies where the loop is still electrically small. At harmonic "H">1, input impedance Z_{IN} (Eq. 83) simplifies to:

SEE EQ. 85 ABOVE



17. The half-circuit concept is applied to understand an efficient differentially driven tapped-capacitor loop antenna.

$$Z_{IN} \approx R_P + \omega^2 M^2 \frac{1}{R_S} \quad (84)$$

$$Z_{inH} \approx R_P + \frac{M^2}{L_S^2} \times R_S + j\omega L_p \quad (85)$$

In the transformer case, current is flowing through the primary and secondary loops. Similar to any current loop, the primary loop is an unavoidable contributor to radiation. Also, except for right around the fundamental frequency, the primary loop exhibits a broadband response with little filtering of the first few harmonics (up to the point where $j\omega L_p$ exerts a pole), unless additional filtering, such as a parallel tank circuit, is used in the driver output. The primary loop can thus dominate over the sec-

$$Z_{inH} = \left(R_{lossPH} + R_{radPH}\right)\frac{M^2}{L_S^2} \times \left(R_{lossSH} + R_{radSH} + R_{cSH}\right) + j\omega L_p \quad (86)$$

$$\frac{P_H}{P_1} = \frac{\eta_H \, i_{rmsH}^2 \, \mathrm{Re}\left(Z_{INH}\right)}{\eta_1 \frac{i_{rms1}^2}{2} R_D} \quad (87)$$

$$\eta_H = \frac{R_{radPH} + \frac{M^2}{L_S^2}\left(R_{radSH}\right)}{\left(R_{lossPH} + R_{radPH}\right) + \frac{M^2}{L_S^2} \times \left(R_{lossSH} + R_{radSH} + R_{cSH}\right)} \quad (88)$$

$$\frac{P_H}{P_1} = 0.632 \times \frac{\eta_H}{\eta_1} \times \frac{\mathrm{Re}\left(Z_{INH}\right)}{R_D} \quad (89)$$

ondary loop as a harmonic radiator, although if the primary area is kept as small as possible (large L_a to obtain necessary mutual inductance, small L_b to keep primary area small) it will normally fall a few decibels under the secondary loop. To calculate R_{rad} for the primary and secondary loops, Eq. 46 (Part 5) must be used for each. Then, Eq. 48 is used twice to calculate loss resistance for both loops. It is then possible to rewrite the input impedance at the harmonics in terms of the primary and secondary resistances of Eq. 85 as:

SEE EQ. 86 ABOVE RIGHT

where:

R_{lossPH} = the primary-loop-series ohmic loss at harmonic H,

R_{radPH} = the primary-loop-series radiation resistance at H,

R_{lossSH} = the secondary-loop-series loss at H,

R_{radSH} = the secondary-loop-radiation resistance at H, and

R_{cSH} = the secondary-tune-capacitor series loss at H.

Assuming that the source resistance ZD >> $j\omega L_p$, it is possible to use the real part of Eq. 86 to write the ratio of radiated harmonic power P_H to carrier power P_1 as:

SEE EQ. 87 ABOVE RIGHT

The harmonic radiation efficiency is provided by:

SEE EQ. 88 ABOVE RIGHT

Further assuming harmonic power to be 10 dB below the carrier and then adding back 5 dB for harmonic directivity, Eq. 87 can be reduced to:

SEE EQ. 89 ABOVE RIGHT

If ZD is not much greater than $j\omega L_p$, then a current divider function may be written similarly to the tapped-capacitor case and added to Eqs. 87 and 89.

The basic performance of the unmatched, tapped capacitor, and inductor antennas with the same sample 12 × 34-mm radiating loop is summarized in Table 7 of Part 5. For the transformer loop, harmonic rejection of 41.5 dB for the second harmonic and 36 dB for the third harmonic was calculated. A 7-dB increase in power between the second and third harmonic was expected due to radiation resistance being a fourth-order function of frequency as in Eq. 45. It is important to note that the harmonic rejection of the transformer-loop antenna is not based on parallel inductive capacitive (LC) filtering, but on extreme mismatching at the harmonic frequency. The loop capacitor brings about the resonance condition in cooperation with the mutual inductance of the transformer, which leads to a good match at the fundamental frequency. Away from the fundamental frequency, this match is not supported and the input impedance of the primary is extremely small, so that i^2R radiated power is also small.

Differential Drivers

Most discrete short-range Tx designs use a single-ended RF output port based on a discrete transistor and are easy to visualize in terms of a driver model referenced to the same ground as RF test instruments. As a result, a single-ended drive has been used in these analyses since it is more illustrative in introducing the basic matching forms. But most integrated Txs use a differential output that is not as intuitively clear. The desire to carry signals in differential mode is a consequence of the need to maintain amplifier stability in the presence of a relatively poor RF ground inside the chip (separated from board ground by bond wire and pin inductances), the need to maintain power supply, ground common-mode noise rejection (the RF circuitry is very close to the digital control circuitry in an integrated Tx), and the convenience of matched devices on the semiconductor die that can meet these needs. A secondary benefit is the extra transmit power that can be provided if voltage swing limits with a single device are the limiting power factor.

The easiest way to visualize differential drivers with a loop antenna is to use the "half-circuit concept" depicted in **Fig. 16**. This concept is based on acknowledging the fact that the drivers are matched but have voltage outputs that are 180 deg. out of phase. This results in points on the circuit where the voltage does not swing relative to ground and these points can be viewed as artificial grounds. This makes it possible to consider the full antenna as consisting of two half circuits that are each driven single ended, and that each remain resonant at the desired frequency with one-half the inductance and resistance, and twice the capacitance of the full circuit. Each half circuit also maintains the same quality factor(Q).

It is not necessary to maintain a perfect geometric balance in a loop antenna to use differential drive. To reduce components parts may be combined. This may result in loop antennas whose functionality is not apparent at a glance, but by breaking the parts back up into the symmetric circuit needed for half-circuit visualization the operation and matching will become clear. For example, the circuit in **Fig. 17** that at first appears to have no matching is seen to actually be the excellent tapped-capacitor form, though highly efficiently implemented with only two physical capacitors.

The authors believe that transformer model of the loop antenna presented here has not been published previously, with the exception of our own recent application note, and that this method for the first time provides a correct basic model of the tapped-loop antenna. Here the circuit designer's intuition can lead to erroneous conclusions and reversion to the underlying EMs is required. Based on the terminal behavior of the loop antenna and its behavior over the first few harmonics where the loop is still electrically small, the relations provided should support approximate prediction of radiated harmonics. However, one often finds that a particular board layout does not meet the predicted harmonic suppression. This is probably most often due to unsuitable effects in the layout, such as harmonic leakage onto power lines that then radiate above the level of the loop. However, it is sometimes due to effects related to the antenna no longer being electrically small. Dealing with these effects falls into the realm of advanced analysis and EM simulation.

The seventh article in this series on short-range radios, to be published this fall, will present detailed practical board-level results with single-ended and differential drivers, and design suggestions for achieving the maximum output power and minimum harmonics at the least cost with practical components and PCB layout methods. These results will be related to the key issue of meeting regulatory requirements. The basics of making approximate engineering laboratory measurements to confirm regulatory compliance will also be reviewed. MRF

FOR FURTHER READING

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